Lecture 10

11. FREQUENCY-SELECTIVE CIRCUITS

11.1. The Series Oscillatory Circuit

- ***11.1.1. Resonance Condition and Basic Circuit Parameters***

The series oscillatory circuit is a circuit that represents a series connection of r-, L-, C-components and an energy source (Fig. 1.9, *a*).

The current in such a circuit is:

$$\dot{I}_m = \frac{\dot{E}_m}{Z} = \frac{\dot{E}_m}{r + jx} = \frac{\dot{E}_m}{r + j\left(\omega L - \frac{1}{\omega C}\right)}.$$

Here $x = \omega L - \frac{1}{\omega C}$ — reactive impedance (reactance) of the circuit.

The phenomenon that causes the circuit impedance to become purely active is called resonance. As it takes place, the circuit reactance is equal to zero

$$x_0 = \omega_0 L - \frac{1}{\omega_0 C} = 0. \quad (11.1)$$

Resonance occurs at the frequency ω_0 from (11.1):

$$\omega_0 = \frac{1}{\sqrt{LC}}. \quad (11.2)$$

Relationship (11.2) is called Thompson's formula. It defines the angular resonance frequency of a circuit.

From (11.2) we get the frequency f_0 and the period T_0:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}; \qquad T_0 = 2\pi\sqrt{LC}.$$

The inductive or capacitive reactance at the resonance frequency is called the wave or characteristic impedance of a circuit:

$$\rho = \omega_0 L = \frac{1}{\omega_0 C}. \qquad (11.3)$$

Taking into account (11.2),

$$\rho = \sqrt{\frac{L}{C}}.$$

The ratio of the characteristic impedance to the active resistance is called the Q-factor of a circuit. Taking into account (11.3)

$$Q = \frac{\rho}{r} = \frac{\omega_0 L}{r} = \frac{1}{\omega_0 r C} = \frac{1}{r}\sqrt{\frac{L}{C}}. \qquad 11.4)$$

Since $\rho >> r$, $Q >> 1$. For radio electronic circuits the Q-factor approaches the values $Q \approx 100\ldots500$. The value inverse to the Q-factor is called attenuation. According to (11.4):

$$d = \frac{1}{Q} = \frac{\rho}{r} = \frac{r}{\omega_0 L} = \omega_0 r C = r\sqrt{\frac{C}{L}}.$$

The values r, L, C of the circuit in Fig. 1.8, *a* are called the primary parameters of the circuit. The values ω_0, f_0, T_0, ρ, Q, d are the secondary parameters of the circuit. The current in the circuit at resonance is

$$\dot{I}_{m0} = \frac{\dot{E}_m}{r}.$$

This is a maximum current and it is limited only by the active resistance of the circuit.

The voltages across the reactive elements of the circuit are:

$$\dot{U}_{mL0} = \dot{I}_{m0}\, j\omega_0 L = \frac{\dot{E}_m}{r} j\omega_0 L = j\frac{\rho}{r}\dot{E}_m = Q\dot{E}_m e^{j\frac{\pi}{2}}; \qquad (11.5)$$

$$\dot{U}_{mC0} = \dot{I}_{m0}\frac{1}{j\omega_0 C} = \frac{\dot{E}_m}{r}\frac{1}{j\omega_0 C} = \frac{1}{j}\frac{\rho}{r}\dot{E}_m = Q\dot{E}_m e^{-j\frac{\pi}{2}}. \qquad (11.6)$$

Since $Q >> 1$, the voltages across the inductance and capacitance at resonance may exceed by dozens and hundreds times the input signal voltage. That's why this kind of resonance is called the voltage resonance.

Obviously, at resonance the vector diagram for a series oscillatory circuit coincides with the diagram in Fig. 6.4. We can see from the vector diagrams and expressions (11.5) and (11.6) that the voltages across the reactive elements of the circuit are equal in amplitude and opposite in phase.

11.1.2. Physical Processes and Basic Energy Relations

The current in a circuit at resonance is:

$$i_0 = I_{m0}\cos\omega_0 t.$$

The energy in the magnetic field of an inductor is:

$$w_{L0} = \frac{Li_0^2}{2} = \frac{LI_{m0}^2}{2}\cos^2\omega_0 t = W_{Lm}\cos^2\omega_0 t, \qquad (11.7)$$

where $W_{Lm} = \dfrac{LI_{m0}^2}{2}$ — maximum energy in the inductor.

The voltage across the capacitance at resonance is:

$$u_{C0} = U_{mC0}\cos\left(\omega_0 t - \frac{\pi}{2}\right) = U_{mC0}\sin\omega_0 t.$$

The energy in the electric field of the capacitor:

$$w_{C_0} = \frac{Cu_{C_0}^2}{2} = \frac{CU_{mC_0}^2}{2}\sin^2\omega_0 t = W_{C_m}\sin^2\omega_0 t, \qquad (11.8)$$

where $W_{C_m} = \dfrac{CU_{mC_0}^2}{2}$ — maximum energy in the capacitor.

Since at resonance the reactive impedance in (11.1) is equal to zero, the reactive power is also equal to zero:

$$Q = 0$$

or

$$I_0^2 x_0 = I_0^2\left(\omega_0 L - \frac{1}{\omega_0 C}\right) = \omega_0 L I_0^2 - \frac{1}{(\omega_0 C)^2} I_0^2 \omega_0 C =$$

$$= \omega_0 L I_0^2 - \omega_0 C U_{C_0}^2 = \omega_0 \frac{L I_{m0}^2}{2} - \omega_0 \frac{C U_{mC_0}^2}{2} =$$

$$= \omega_0 W_{L_m} - \omega_0 W_{C_m} = \omega_0 \left(W_{L_m} - W_{C_m}\right) = 0.$$

Hence

$$W_{L_m} - W_{C_m} = 0, \quad W_{L_m} = W_{C_m} = W_m. \tag{11.9}$$

It means that the maximum energies, stored in the inductance and the capacitance, are identical.

The total energy in the circuit according to (11.7), (11.8) and (11.9):

$$w_0 = w_{L0} + w_{C0} = W_{Lm}\cos^2\omega_0 t + W_{Cm}\sin^2\omega_0 = W_m.$$

Therefore, the total energy in the circuit remains constant at any instant of time. It oscillates only between the magnetic field of the inductor and that of the capacitor (Fig. 11.1, *a*, *b*).

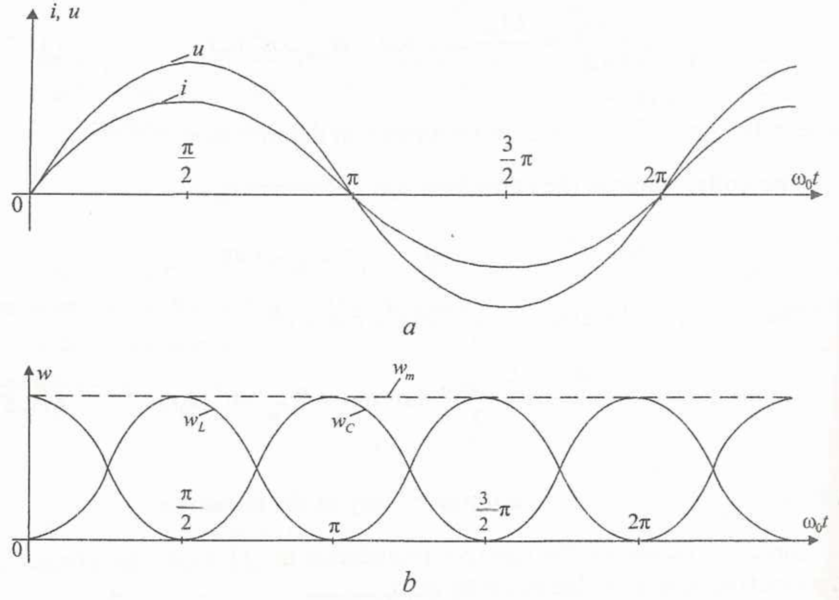

Fig. 11.1

The frequency of the energy oscillations is equal to $2\omega_0$, for example, for the inductance from (11.7):

$$w_{L0} = W_{Lm}\cos^2\omega_0 t = W_{Lm}\frac{1+2\cos 2\omega_0 t}{2} = \frac{W_{Lm}}{2} + W_{Lm}\cos 2\omega_0 t.$$

According to (11.4), the Q-factor is

$$Q = \frac{\omega_0 L}{r} = \omega_0 \frac{L I_0^2}{r I_0^2} = \frac{2\pi}{T_0}\frac{W_m}{P} = 2\pi\frac{W_m}{W_r},$$

where W_r is the energy, dissipated by the active resistance of the circuit. Therefore, the Q-factor is defined as the ratio of the energy stored in the reactive circuit elements to the energy of losses in its active resistance.

11.1.3. Frequency Characteristics of a Series Oscillatory Circuit

Analyzing the frequency characteristics of a series oscillatory circuit, we will use the circuit complex functions described in Chapter 10. Consider the complex input admittance $Y(j\omega)$ in Fig. 10.8, *a*

$$Y(j\omega) = \frac{I_m(j\omega)}{E_m(j\omega)} = \frac{1}{Z(j\omega)} = \frac{1}{r+jx} = \frac{1}{r\left(1+j\dfrac{x}{r}\right)}. \tag{11.10}$$

Designate the generalized circuit detuning as

$$\xi = \frac{x}{r} = \tan\varphi. \tag{11.11}$$

Then from (11.10)

$$Y(j\omega) = \frac{1}{r(1+j\xi)}.$$

At the resonance frequency $\omega = \omega_0$, then $x = 0$ and $\xi = 0$, and we get:

$$Y(j\omega) = \frac{1}{r}.$$

When comparing circuits it is convenient to use the normalized input conductance:

$$Y_n(j\omega)=Y_n(\omega)e^{-j\varphi(\omega)}=\frac{Y(j\omega)}{Y(j\omega_0)}=\frac{1}{1+j\xi},$$

where $Y_n(\omega)=\frac{1}{\sqrt{1+\xi^2}}$ — normalized amplitude-frequency characteristic; $\varphi(\omega)=\text{atan}\xi$ — phase-frequency characteristic (PhFC).

The analysis of the frequency characteristics of a series oscillatory circuit shows that even a small deviation of the input signal frequency ω from the resonance frequency ω_0 causes significant changes in the amplitude and phase of the complex input admittance (AFC and PhFC). Therefore, it is convenient to introduce the concept of circuit detuning.

The difference between the input signal source frequency and the resonance frequency is called the absolute detuning:

$$\Delta\omega=\omega-\omega_0.$$

The ratio

$$\delta=\frac{\Delta\omega}{\omega_0}=\frac{\omega-\omega_0}{\omega_0}$$

is called the relative detuning. The value ξ from (11.11)

$$\xi=\frac{X}{r}=\frac{1}{r}\left(\omega L-\frac{1}{\omega C}\right)=\frac{1}{r}\left(\omega_0 L\frac{\omega}{\omega_0}-\frac{1}{\omega_0 C}\frac{\omega_0}{\omega}\right)= \\ =\frac{1}{r}\cdot\left(\rho\frac{\omega}{\omega_0}-\rho\frac{\omega_0}{\omega}\right)=\frac{\rho}{r}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)=Q\nu \qquad (11.12)$$

is called the generalized circuit detuning of the since it characterizes the detuning caused both by the frequency deviation of the input signal ω and by the changes in the circuit parameters (in terms of the Q-factor).

Here $\nu=\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}$ — detuning factor.

It is obvious that

$$\nu=\frac{\omega^2-\omega_0^2}{\omega\omega_0}=\frac{(\omega-\omega_0)(\omega+\omega_0)}{\omega\omega_0}\approx\frac{2\omega\Delta\omega}{\omega\omega_0}=\frac{2\Delta\omega}{\omega_0}=2\delta. \qquad (11.13)$$

From (11.12), taking into account (11.13), we get:

$$\xi=2\delta Q.$$

Taking into account (11.12) and (11.13), the AFC and PhFC can be written as:

$$Y_n(\omega)=\frac{1}{\sqrt{1+Q^2\nu^2}}=\frac{1}{\sqrt{1+Q^2\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)^2}}\approx\frac{1}{\sqrt{1+4Q^2\delta^2}};$$

$$\varphi(\omega)=\text{atan}Q\nu=\text{atan}Q\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)\approx\text{atan}2Q\delta.$$

The frequency characteristics of a series oscillatory circuit are given in Fig. 11.2, *a*, *b*. Here $Q_1>Q_2>Q_3$.

Since $\varphi(\omega)$ defines the phase angle of the circuit input resistance, at frequencies below the resonance one the capacitive reactance becomes greater than the inductive reactance, and the circuit becomes to be of capacitive nature ($\varphi<0$). At $\omega=0$ this reactance becomes purely capacitive $\left(\varphi\to-\frac{\pi}{2}\right)$, and at resonance frequency the circuit has purely active resistance. At frequencies higher than the resonance frequency, the inductive reactance of the circuit is higher than its capacitive reactance ($\varphi>0$) and at $\omega\to\infty$ it tends to be purely inductive $\left(\varphi\to\frac{\pi}{2}\right)$.

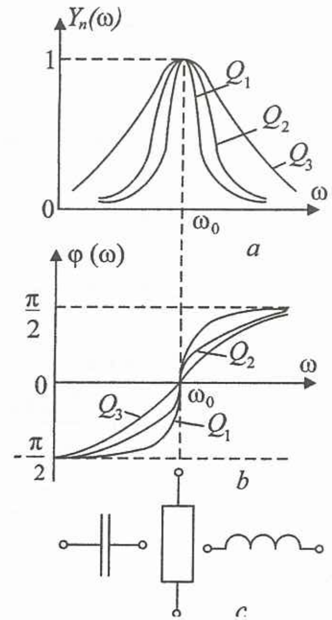

Fig. 11.2

Consider the complex voltage transfer ratios.

For the active resistance:

$$K_{ur}(j\omega)=\frac{U_{mr}(j\omega)}{E_m(j\omega)}=\frac{rI_m(j\omega)}{E_m(j\omega)}=rY(j\omega)$$

or

$$K_{ur}(j\omega)=K_{ur}(\omega)e^{j\varphi_r(\omega)}=\frac{Y(j\omega)}{Y(j\omega_0)}=Y_n(j\omega)=\frac{1}{1+j\xi},$$

where

$$K_{ur}(\omega)=\frac{1}{\sqrt{1+\xi^2}}=\frac{1}{\sqrt{1+Q^2\nu^2}}=\frac{1}{\sqrt{1+Q^2\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)^2}}\approx\frac{1}{\sqrt{1+4Q^2\delta^2}}; \quad (11.14)$$

$$\varphi_r(\omega)=-\operatorname{atan}\xi=-\operatorname{atan}Q\nu=-\operatorname{atan}Q\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)\approx-\operatorname{atan}2Q\delta. \quad (11.15)$$

At $\omega=0,\ K_{ur}(0)=0,\quad \varphi_r(0)=\frac{\pi}{2}.$

At $\omega=\omega_0,\ K_{ur}(\omega_0)=1,\quad \varphi_r(\omega_0)=0.$

At $\omega\to\infty,\ K_{ur}(\infty)\to 0,\quad \varphi_r(\infty)\to -\frac{\pi}{2}.$

For the inductance:

$$K_{uL}(j\omega)=\frac{U_{mL}(j\omega)}{E_m(j\omega)}=\frac{I_m(j\omega)\,j\omega L}{E_m(j\omega)}=Y(j\omega)\,j\omega L=\frac{rY(j\omega)\,j\omega_0 L\omega}{r\,\omega_0}$$

or

$$K_{uL}(j\omega)=K_{uL}(\omega)e^{j\varphi_L(\omega)}=Y_n(j\omega)\,jQ\frac{\omega}{\omega_0}=\frac{jQ}{1+j\xi}\frac{\omega}{\omega_0};$$

$$K_{uL}(\omega)=\frac{Q}{\sqrt{1+\xi^2}}\frac{\omega}{\omega_0}=\frac{Q}{\sqrt{1+Q^2\nu^2}}\frac{\omega}{\omega_0}=\frac{Q}{\sqrt{1+Q^2\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)^2}}\frac{\omega}{\omega_0}\approx\frac{Q}{\sqrt{1+4Q^2\delta^2}}\frac{\omega}{\omega_0}; \quad (11.16)$$

$$\varphi_L(\omega)=\frac{\pi}{2}-\operatorname{atan}\xi=\frac{\pi}{2}-\operatorname{arctg}Q\nu=\frac{\pi}{2}-\operatorname{atan}Q\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)\approx\frac{\pi}{2}-\operatorname{atan}2Q\delta. \quad (11.17)$$

At $\omega=0,\ K_{uL}(0)=0,\quad \varphi_L(0)=\pi.$

At $\omega=\omega_0,\ K_{uL}(\omega_0)=Q,\quad \varphi_L(\omega_0)=\frac{\pi}{2}.$

At $\omega\to\infty,\ K_{uL}(\infty)\to 1,\quad \varphi_L(\infty)\to 0.$

For the capacity:

$$K_{uC}(j\omega)=\frac{U_{mC}(j\omega)}{E_m(j\omega)}=\frac{I_m(j\omega)}{j\omega C E_m(j\omega)}=\frac{Y(j\omega)}{j\omega C}=\frac{rY(j\omega)}{rj\omega C}\frac{\omega_0}{\omega}$$

or

$$K_{uC}(j\omega)=K_{uC}(\omega)e^{j\varphi_C(\omega)}=Y_n(j\omega)Q\frac{\omega_0}{\omega}=\frac{Q}{1+j\xi}\frac{\omega_0}{j\omega},$$

where

$$K_{uC}(\omega)=\frac{Q}{\sqrt{1+\xi^2}}\frac{\omega_0}{\omega}=\frac{Q}{\sqrt{1+Q^2\nu^2}}\frac{\omega_0}{\omega}=\frac{Q}{\sqrt{1+Q^2\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)^2}}\frac{\omega_0}{\omega}\approx\frac{Q}{\sqrt{1+4Q^2\delta^2}}\frac{\omega_0}{\omega}; \quad (11.18)$$

$$\varphi_C(\omega) = -\frac{\pi}{2} - \operatorname{atan}\xi = -\frac{\pi}{2} - \operatorname{atan}Q\nu =$$
$$= -\frac{\pi}{2} - \operatorname{atan}Q\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) \approx -\frac{\pi}{2} - \operatorname{atan}2Q\delta. \qquad (11.19)$$

At $\omega = 0$, $K_{uC}(0) = 1$, $\varphi_C(0) = 0$.

At $\omega = \omega_0$, $K_{uC}(\omega_0) = Q$, $\varphi_C(\omega_0) = -\frac{\pi}{2}$.

At $\omega \to \infty$, $K_{uC}(\infty) \to 0$, $\varphi_C(\infty) \to -\pi$.

Build the amplitude-frequency characteristics according to (11.14), (11.16), (11.18) and the phase-frequency characteristics according to (11.15), (11.17), (11.19) (Fig. 11.3).

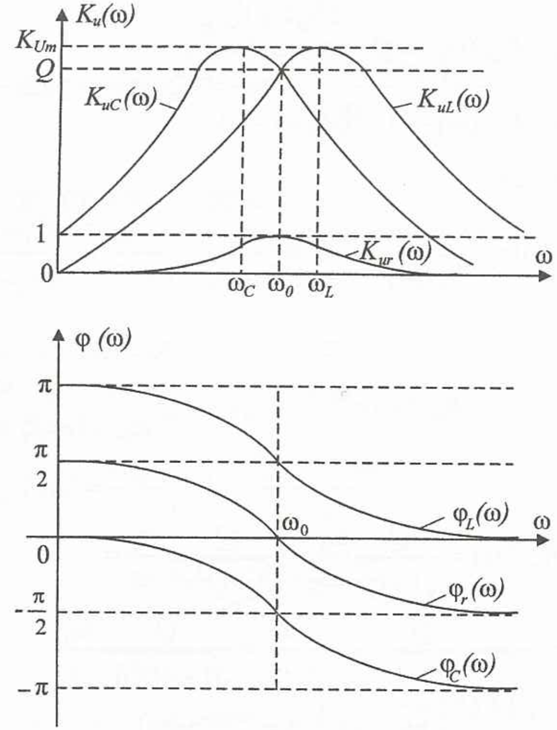

Fig. 11.3

Analyzing the functions $K_{uL}(\omega)$, $K_{uC}(\omega)$ in (11.16) and (11.18) we get the values of the frequencies ω_L and ω_c at which the transfer ratios reach their maximum values that exceed Q:

$$\omega_L = \omega_0\sqrt{\frac{2Q^2}{2Q^2-1}}, \qquad \omega_C = \omega_0\sqrt{\frac{2Q^2-1}{2Q^2}}.$$

As it takes place,

$$K_{uL_m}(\omega_L) = K_{uC_m}(\omega_C) = \frac{2Q^2}{\sqrt{4Q^2-1}}.$$

The higher the Q-factor of the circuit, the smaller the difference is between ω_L and ω_C on the one side and ω_0 — on the other as well as the difference between $K_{uLm}(\omega_L) = K_{uCm}(\omega_C)$ and Q. That is why for high-Q circuits it is assumed that

$$\omega_L \approx \omega_C \approx \omega_0,\ K_{uL_m}(\omega_L) = K_{uC_m}(\omega_C) = Q$$

and the resonance is judged by the maximum voltage across the capacitor.

11.1.4. The Concepts of Selectivity, Bandwidth, and Bandwidth Shape Factor

The ability of a circuit to select signals of a definite frequency range from signals of other frequencies is called selectivity.

The bandwidth of a series oscillatory circuit is a frequency interval around the resonance frequency at the boundaries of which the current in the circuit reduces to $\frac{1}{\sqrt{2}} = 0{,}707$ of its maximum (resonance) value.

The difference between the boundaries of the frequency range is called the absolute bandwidth (Fig. 11.4):

$$B = \omega_2 - \omega_1.$$

The relative bandwidth $B_0 = \frac{B}{\omega_0} = \frac{\omega_2 - \omega_1}{\omega_0}$.

Determine the bandwidth B (L) in Fig. 11.4.

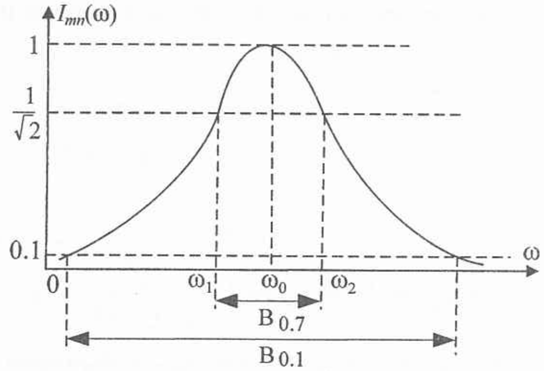

Fig. 11.4

Since the modulus of the normalized current in a circuit coincides with the amplitude-frequency characteristic of the complex input conductance, the frequencies ω_1 and ω_2 can be determined from the expression:

$$I_{mn}(\omega) = \frac{1}{\sqrt{1+\xi^2}} = \frac{1}{\sqrt{2}}.$$

Hence

$$\xi_{1,2} = \pm 1$$

or

$$\begin{cases} \xi_1 = Q\left(\dfrac{\omega_1}{\omega_0} - \dfrac{\omega_0}{\omega_1}\right) = -1, \\ \xi_2 = Q\left(\dfrac{\omega_2}{\omega_0} - \dfrac{\omega_0}{\omega_2}\right) = 1. \end{cases}$$

As a result we obtain the following equation system:

$$\begin{cases} \omega_1^2 + \dfrac{1}{Q}\omega_0\omega_1 - \omega_0^2 = 0, \\ \omega_2^2 - \dfrac{1}{Q}\omega_0\omega_2 - \omega_0^2 = 0. \end{cases} \qquad (11.20)$$

Solving the system (11.20) and taking into account only the positive frequencies we get:

$$\begin{cases} \omega_1 = \omega_0\left[-\dfrac{1}{2Q} + \sqrt{1+\left(\dfrac{1}{2Q}\right)^2}\right], \\ \omega_2 = \omega_0\left[\dfrac{1}{2Q} + \sqrt{1+\left(\dfrac{1}{2Q}\right)^2}\right]. \end{cases}$$

Then the absolute bandwidth is:

$$B = \frac{\omega_0}{Q}. \qquad (11.21)$$

The relative bandwidth is:

$$B_0 = \frac{1}{Q} = d.$$

That is, the relative bandwidth thus determined is equal to the attenuation factor of the circuit.

The ideal circuit passes all frequencies within its bandwidth and does not pass frequencies beyond it. The amplitude-frequency characteristic of such a circuit has the form of a rectangle (Fig. 11.5).

The degree of approximation of a real frequency characteristic (see Fig. 11.4) to the ideal one is evaluated by the bandwidth shape factor (ratio).

$$K_B = \frac{B_{0,707}}{B_{0,1}}. \qquad (11.22)$$

where $B_{0,707}$, $B_{0,1}$ are bandwidths defined at the levels $\frac{1}{\sqrt{2}} = 0,707$ and 0,1 of the maximum value of the AFHC.

In Fig. 11.5 we can see that for the ideal circuit $K_B = 1$.

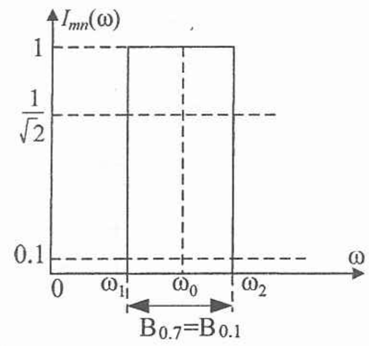

Fig. 11.5

Determine $B_{0.1}$ considering the equation $\dfrac{1}{\sqrt{1+\xi^2}} = 0{,}1$.

We get $B_{0{,}1} = \dfrac{10\omega_0}{Q}$.

Then, taking account of (11.21) and (11.22), from (11.22) we obtain $K_B = 0{,}1$. Therefore, the bandwidth shape factor for series oscillatory circuits of all kinds is the same and does not depend on their Q-factors.

It is clear that the selectivity of a real circuit ($K_B = 0{,}1$) is quite low as compared with the ideal circuit ($K_B = 1$).

11.1.5. Some Applications of Series Oscillatory Circuits

The input circuit of a radio receiver (Fig. 11.6) can serve an example of series oscillatory circuit application. Here, an input signal comes to the LC-circuit from the receiving aerial due to the magnetic coupling M. The EMF of the signal is, therefore, in series with L, C and the active resistance r (the resistances of the inductor L and of the connecting wires). The signal from the capacitor C comes to the input of the transistor of a high-frequency amplifier. Therefore, the input circuit of the radio receiver is a series oscillatory circuit (Fig. 11.7).

Another example of series oscillatory circuit application is a filter in which several series oscillatory circuits at the resonance frequencies ω_{01}, ω_{02}, ω_{03} bypass the load r_l (Fig. 11.8). At these frequencies resonances occur in the circuits, and signals of these frequencies do not arrive at the load r_l from the source $e(t)$ and develops completely across the internal ballast resistor R_i.

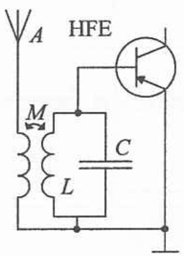

Fig. 11.6

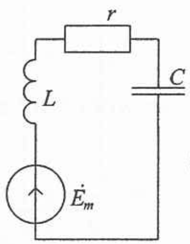

Fig. 11.7

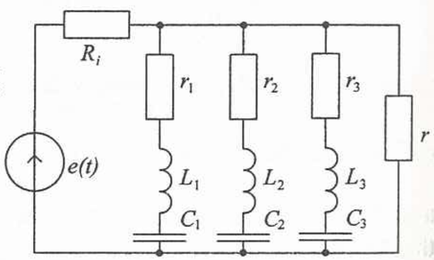

Fig. 11.8

The signal with a frequency different from ω_1, ω_2, ω_3. gets freely to the load since at these frequencies the circuits have greater resistances and do not exert any by-pass effect on the signal. For greater filtration efficiency, the following relationships should be observed: $R_1 >> r_1$, $R_1 >> r_2$, $R_1 >> r_3$.

Example 1

How will the resonance frequency ω_0, quality factor Q and bandwidth $2\Delta\omega_0$ of a series oscillatory circuit change if the following parameters increase 2-fold:

a) loss resistance R;
b) inductance Z;
c) capacitance C?

Solution

The expressions for the resonance frequency ω_0, quality factor Q and bandwidth $2\Delta\omega_0$ are written as:

$$\omega_0 = \frac{1}{\sqrt{LC}};\; Q = \frac{\rho}{R} = \frac{1}{R}\sqrt{\frac{L}{C}};\; 2\Delta\omega_0 = \frac{\omega_0}{Q} = \frac{R\sqrt{C}}{\sqrt{LC}\sqrt{L}} = \frac{R}{L}.$$

Then

1) if the loss resistance R increase 2-fold:
   ω_0 — does not change;
   Q — decreases 2-fold;
   $2\Delta\omega_0$ — increases 2-fold;

2) if the inductance L increases 2-fold:
   ω_0 - decreases 2-fold;
   Q — increases 2-fold;
   $2\Delta\omega_0$ — decreases 2-fold;

3) if the capacitance C increases 2-fold:
   ω_0 — decreases 2-fold;
   Q — decreases 2-fold;
   $2\Delta\omega_0$ — does not change.